# **Consistency Models**



CS 240: Computing Systems and Concurrency Lecture 13

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#### **Consistency Models**

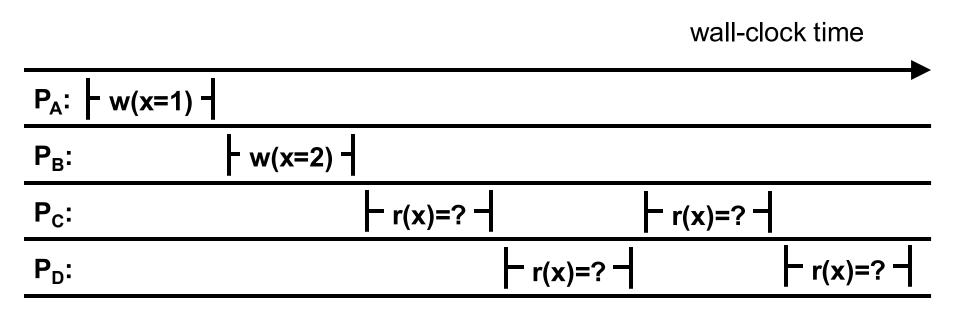
- Contract between a distributed system and the applications that run on it
- A consistency model is a set of guarantees made by the distributed system
- We are concerned with: "what happens if a client modifies some data items and concurrently another client reads or modifies the same items possibly at a different replica"?

#### Linearizability [Herlihy and Wing 1990]

- All replicas execute operations in some total order
- That total order preserves the real-time ordering between operations
  - If operation A completes before operation B begins, then A is ordered before B in real-time
  - If neither A nor B completes before the other begins, then there is no real-time order
    - (But there must be *some* total order)

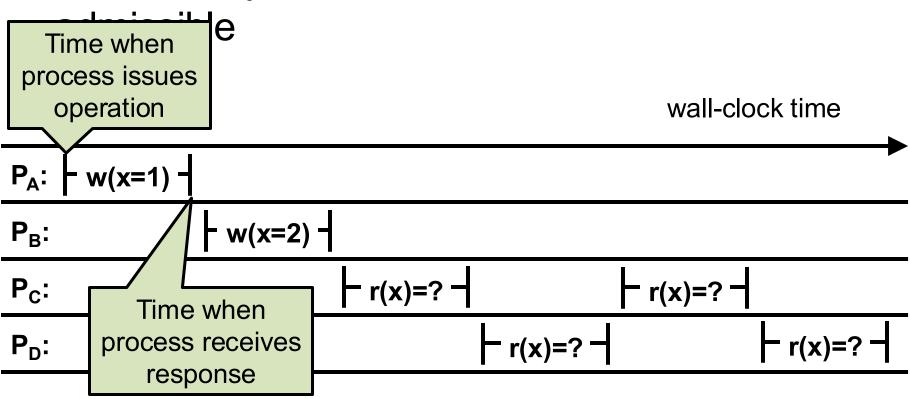
## Intuitive example

Consistency model defines what values reads are admissible



#### Intuitive example

Consistency model defines what values reads are



## Linearizability

- Any execution is the same as if all read/write ops were executed in order of wall-clock time at which they were issued
- Therefore:
  - Reads are never stale (i.e., a read returns the value that was last written)
  - All replicas enforce wall-clock ordering for all writes

$$P_A$$
:  $| w(x=1) |$ 
 $P_B$ :  $| w(x=2) |$ 
 $P_C$ :  $| r(x)=? |$ 
 $| r(x)=? |$ 
 $| r(x)=? |$ 

## Linearizability: YES

- Any execution is the same as if all read/write ops were executed in order of wall-clock time at which they were issued
- Therefore:
  - Reads are never stale (i.e., a read returns the value that was last written)
  - All replicas enforce wall-clock ordering for all writes

$$P_A$$
:  $| w(x=1) |$ 
 $P_B$ :  $| w(x=2) |$ 
 $P_C$ :  $| r(x)=2 |$ 
 $| r(x)=2 |$ 
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 $| r(x)=2 |$ 

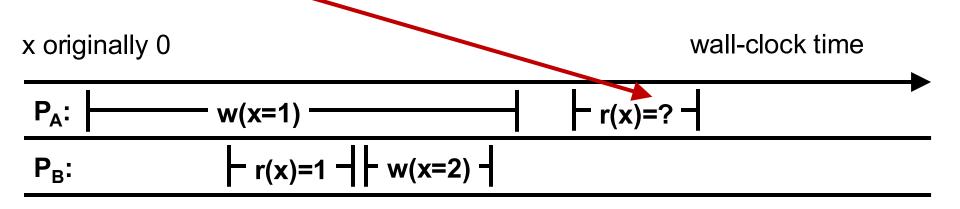
## **Linearizability: NO**

- Any execution is the same as if all read/write ops were executed in order of wall-clock time at which they were issued
- Therefore:
  - Reads are never stale (i.e., a read returns the value that was last written)
  - All replicas enforce wall-clock ordering for all writes

$$P_A$$
:  $| w(x=1) |$ 
 $P_B$ :  $| w(x=2) |$ 
 $P_C$ :  $| r(x)=1 |$ 
 $| r(x)=2 |$ 
 $P_D$ :  $| r(x)=2 |$ 

#### Linearizability: Quiz

 If the execution is linearizable, what does P<sub>A</sub> read here?



 $P_A$  sees the latest write that took effect on the system (x=2)

# Linearizability == "Appears to be a Single Machine"

- Single machine processes requests one by one in the order it receives them
  - Will receive requests ordered by real-time in that order
  - Will receive all requests in some order
- Atomic Multicast, Viewstamped Replication, Paxos, and RAFT provide Linearizability
- Single machine processing incoming requests one at a time also provide Linearizability ©

#### Linearizability is ideal?

- Hides the complexity of the underlying distributed system from applications!
  - Easier to write applications
  - Easier to write correct applications
- But, performance trade-offs

#### Stronger vs weaker consistency

- Stronger consistency models
  - + Easier to write applications
  - More guarantees for the system to ensure
     Results in performance trade-offs
- Weaker consistency models
  - Harder to write applications
  - + Fewer guarantees for the system to ensure

#### Strictly stronger consistency

- A consistency model A is strictly stronger than B if it allows a strict subset of the behaviors of B
  - Guarantees are strictly stronger

#### Sequential consistency

All replicas execute operations in some total order

- That total order preserves the process ordering between operations
  - If process P issues operation A before operation B, then A is order before B by the process order
  - If operations A and B are done by different processes then there is no process order between them
    - (But there must be some total order)

# Sequential Consistency ≈ "Appears to be a Single Machine"

- Single machine processes requests one by one in the order it receives them
  - Will receive requests ordered by process order in that order
  - Will receive all requests in some order

# Linearizability is strictly stronger than Sequential Consistency

- Linearizability: ∃total order + real-time ordering
- Sequential: ∃total order + process ordering
  - Process ordering ⊆ Real-time ordering

#### Sequential consistency

- Sequential = Linearizability real-time ordering
  - 1. All servers execute all ops in *some* identical sequential order
  - 2. Global ordering preserves each client's own local ordering

- With concurrent ops, "reordering" of ops (w.r.t. realtime ordering) acceptable, but all servers must see same order
  - e.g., linearizability cares about time sequential consistency cares about program order

## Sequential consistency

- Any execution is the same as if all read/write ops were executed in some global ordering, and the ops of each client process appear in the program order
- Therefore:
  - Reads may be stale in terms of real time, but not in logical time
  - Writes are totally ordered according to logical time across all replicas

 $P_A$ : | w(x=1) | 

  $P_B$ : | w(x=2) | 

  $P_C$ : | r(x)=? | | r(x)=? | 

  $P_D$ : | r(x)=? | 

## Sequential consistency: YES

- Any execution is the same as if all read/write ops were executed in some global ordering, and the ops of each client process appear in the program order
- Therefore:
  - Reads may be stale in terms of real time, but not in logical time
  - Writes are totally ordered according to logical time across all replicas

P<sub>A</sub>: | w(x=1) |

P<sub>B</sub>: | w(x=2) |

P<sub>C</sub>: | r(x)=2 | | r(x)=2 |

P<sub>D</sub>: | r(x)=2 |

Also valid with linearizability

# Sequential consistency: YES

- Any execution is the same as if all read/write ops were executed in some global ordering, and the ops of each client process appear in the program order
- Therefore:
  - Reads may be stale in terms of real time, but not in logical time
  - Writes are totally ordered according to logical time across all replicas

Not valid with linearizability

## Sequential consistency: NO

- Any execution is the same as if all read/write ops were executed in some global ordering, and the ops of each client process appear in the program order
- Therefore:
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  - Writes are totally ordered according to logical time across all replicas

 $P_A$ : | w(x=1) | 

  $P_B$ : | w(x=2) | 

  $P_C$ : | r(x)=2 | 

  $P_C$ : | r(x)=1 | 

  $P_C$ : | r(x)=1 |

No global ordering can explain these results

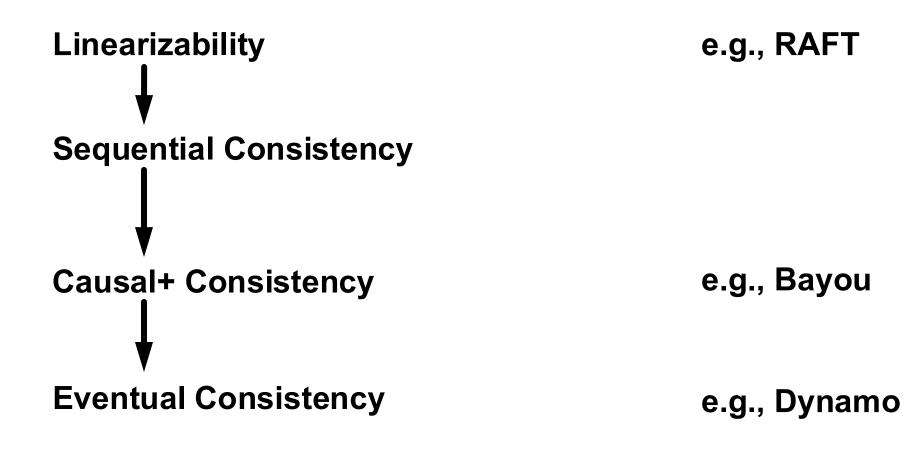
## Sequential consistency: NO

- Any execution is the same as if all read/write ops were executed in some global ordering, and the ops of each client process appear in the program order
- Therefore:
  - Reads may be stale in terms of real time, but not in logical time
  - Writes are totally ordered according to logical time across all replicas

 $P_{A}$ : w(x=1) w(x=3)  $P_{B}$ : w(x=2)  $P_{C}$ : r(x)=3 r(x)=1 r(x)=1 r(x)=2

No sequential global ordering can explain these results... E.g.: w(x=3), r(x)=3, r(x)=1, w(x=2) doesn't preserve  $P_A$ 's ordering

# **Consistency hierarchy**



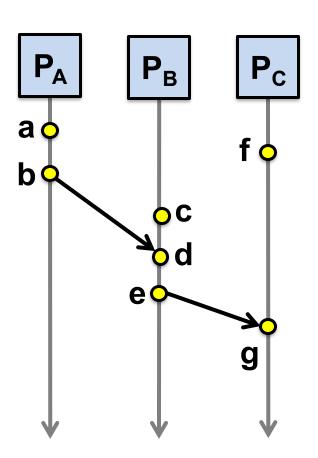
- Partially orders all operations, does not totally order them
  - Does not look like a single machine

#### Guarantees

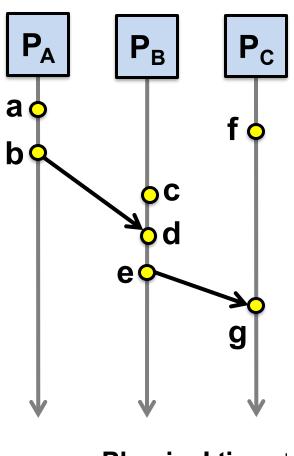
- For each process, ∃ an order of all writes + that process's reads
- Order respects the happens-before (→) ordering of operations
- + in Causal+ means replicas converge to the same state
  - Skip details, makes it stronger than eventual consistency

- Writes that are potentially causally related must be seen by all processes in same order
- 2. Concurrent writes may be seen in a different order on different processes
- Concurrent: Ops not causally related

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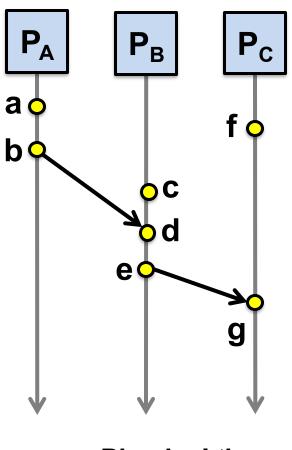


Operations	Concurrent?
a, b	
b, f	
c, f	
e, f	
e, g	
a, c	
a, e	



Physical time ↓

Operations	Concurrent?
a, b	N
b, f	Υ
c, f	Υ
e, f	Υ
e, g	N
a, c	Υ
a, e	N



Physical time ↓

## Causal+ But Not Sequential

$$P_{A} \vdash w(x=1) \dashv \vdash r(y)=0 \dashv$$

$$P_{B} \vdash w(y=1) \dashv \vdash r(x)=0 \dashv$$

#### √ Casual+

Happens  $w(x=1) \rightarrow r(y)=0$ Before Order  $w(y=1) \rightarrow r(x)=0$ 

P<sub>A</sub> Order: w(x=1), r(y=0), w(y=1) ■

P<sub>B</sub> Order: w(y=1), r(x=0), w(x=1)

#### **X** Sequential

Process 
$$w(x=1) \rightarrow r(y)=0$$
  
Ordering  $w(y=1) \rightarrow r(x)=0$ 

$$w(x=1) \rightarrow r(y)=0$$
No Total
Order  $w(y=1) \rightarrow r(x)=0$ 

#### **Eventual But Not Causal+**

$$P_A \vdash w(x=1) \vdash w(y=1) \vdash$$

 $P_B$ 

#### √ Eventual

As long as P<sub>B</sub>
eventually would
see r(x)=1 this is
fine

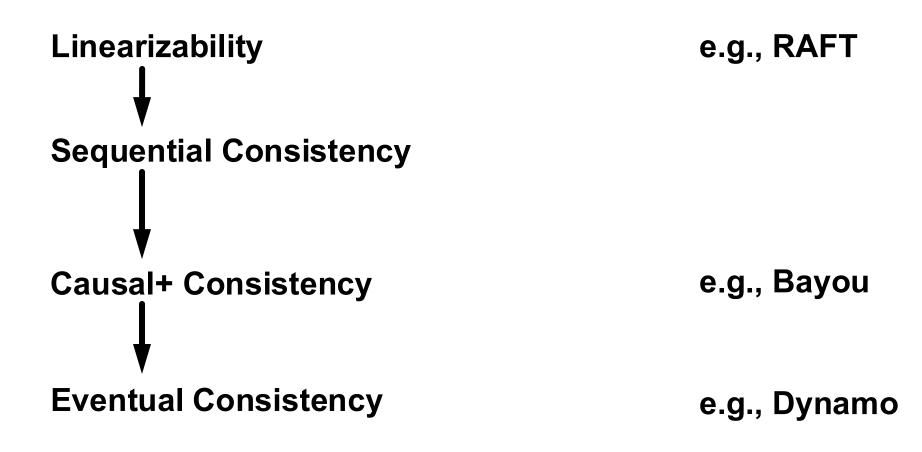
$$|-r(y)=1-|-r(x)=0-|$$

#### X Causal+

Happens 
$$w(x=1) \rightarrow w(y)=1$$
Before  $\forall$ 
Ordering  $r(y)=1 \rightarrow r(x)=0$ 

No Order for 
$$P_B$$
  $r(y)=1 \longrightarrow r(x)=0$ 

# **Summary: Consistency hierarchy**



## Causal Consistency: Quiz

$$P_{A} \mid w(x=1) \mid w(x=3) \mid w(x=3) \mid x=2 \mid x=2 \mid x=3 \mid x=3 \mid x=2 \mid x=3 \mid$$

- Valid under causal consistency
- Why? x=3 and x=2 are concurrent
  - So all processes don't (need to) see them in same order
- P<sub>C</sub> and P<sub>D</sub> read the values '1' and '2' in order as potentially causally related. No 'causality' for '3'.

# Sequential Consistency: Quiz

$$P_{A} \vdash w(x=1) \dashv \qquad \qquad \vdash w(x=3) \dashv \qquad \qquad \qquad \qquad \vdash r(x)=1 \dashv \vdash w(x=2) \dashv \qquad \qquad \qquad \qquad \qquad \vdash r(x)=3 \dashv \vdash r(x)=2 \dashv \qquad \qquad \qquad \qquad \vdash r(x)=2 \dashv \vdash r(x)=3 \dashv \qquad \qquad \qquad \vdash r(x)=3 \dashv \qquad \vdash r(x)=3 \dashv \qquad \vdash r(x)=3 \dashv \qquad \vdash r(x)=3 \dashv \qquad \qquad \vdash r(x)=3 \dashv \qquad \vdash r(x)$$

- Invalid under sequential consistency
- Why? P<sub>C</sub> and P<sub>D</sub> see 2 and 3 in different order
- But fine for causal consistency
  - 2 and 3 are not causally related

$$P_{A} \vdash w(x=1) \dashv P_{B} \vdash r(x)=1 \dashv \vdash w(x=2) \dashv P_{C} \vdash r(x)=1 \dashv r($$

x=2 happens after x=1

$$P_{A} \mid w(x=1) \mid P_{B} \mid w(x=2) \mid P_{C} \mid r(x)=2 \mid r(x)=1 \mid r(x)=2 \mid r(x)$$



P<sub>B</sub> doesn't read value of 1 before writing 2

#### Visualization of linearizability ©

 Nice way to see and think when a certain execution is / isn't allowed in linearizability

https://mwhittaker.github.io/consistency in distributed systems/2 cap.html

Also check out:

https://mwhittaker.github.io/blog/visualizing\_linearizability/

https://muratbuffalo.blogspot.com/2021/10/linearizability.html